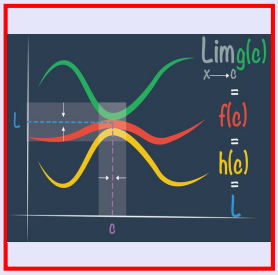
Math 261

Spring 2023

Lecture 49



Feb 19-8:47 AM

If $f(x)$ is cont. on $[a,b]$, then

average value of $f(x)$ on $[a,b]$ is

$$\boxed{f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx}$$

Class QZ 13:

Find the average value of $\boxed{f(x)=\sqrt[3]{x}}$ on $[1,8]$. Exact answer required.

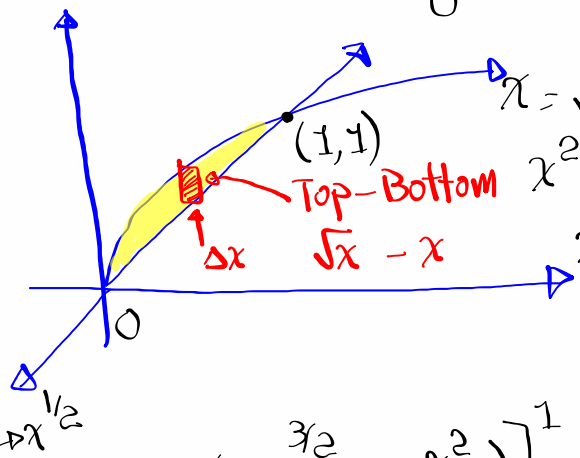
$a=1,\ b=8$

$$f_{ave} = \frac{1}{8-1}\int_1^8 \sqrt[3]{x}\,dx = \frac{1}{7}\int_1^8 x^{1/3}\,dx = \frac{1}{7}\cdot\frac{x^{4/3}}{4/3}\Big]_1^8$$

$$= \frac{3}{28}\left(8^{4/3} - 1^{4/3}\right) = \frac{3}{28}(16-1) = \frac{3}{28}\cdot 15 = \boxed{\frac{45}{28}}$$

May 11-9:24 AM

Find the area enclosed by $f(x)=\sqrt{x}$ and $g(x)=x$.



$x=\sqrt{x}$

$x^2=x$

$x^2-x=0$

$x(x-1)=0$

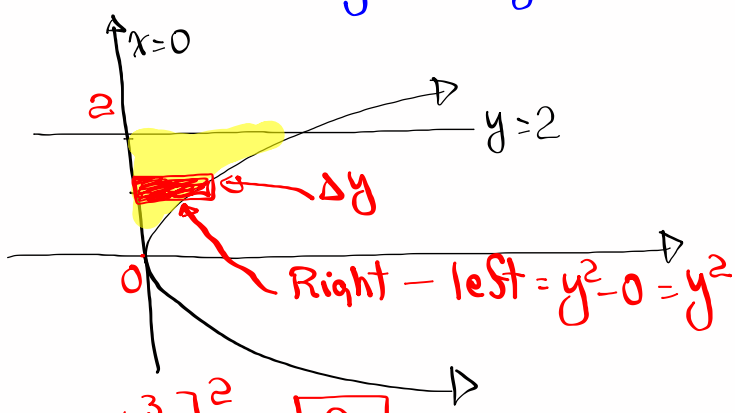
$x=0\ ,\ x=1$

$$A=\int_0^1 (\sqrt{x}-x)\,dx = \left(\frac{x^{3/2}}{3/2}-\frac{x^2}{2}\right)\Bigg]_0^1$$

$$=\frac{2}{3}\cdot 1^{3/2}-\frac{1}{2}\cdot 1^2=\frac{2}{3}-\frac{1}{2}=\boxed{\frac{1}{6}}$$

May 15-8:49 AM

Find the area enclosed by $x=y^2$, $x=0$, and $y=2$.

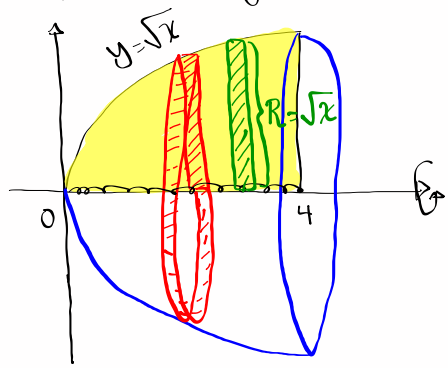


$$A=\int_0^2 y^2\,dy=\frac{y^3}{3}\Bigg]_0^2=\boxed{\frac{8}{3}}$$

May 15-8:55 AM

Now finding volume of revolution:

Consider the region shaded below



we want to rotate this region by x-axis

Ref. Rec. ⊥ Axis of Revolution

Region is attached to the axis of Revolution

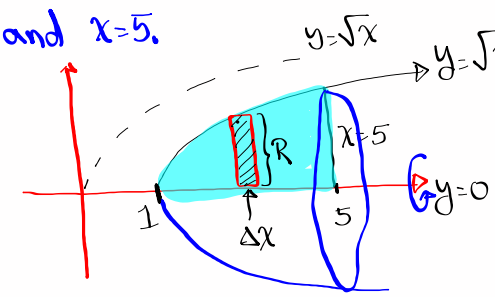
Disk method

$$\int_a^b \pi R^2\,dx \quad \text{or} \quad \int_c^d \pi R^2\,dy$$

$$V=\int_0^4 \pi(\sqrt{x})^2\,dx=\pi\int_0^4 x\,dx=\pi\cdot\frac{x^2}{2}\Big]_0^4=\boxed{8\pi}$$

May 15-8:59 AM

Draw the enclosed region by $y=\sqrt{x-1}$, $y=0$, and $x=5$.



1) Ref. Rec. ⊥ A.O.R.

2) Region attached to A.O.R.

Disk Method

Rotate about x-axis.

$$\int_1^5 \pi(\sqrt{x-1})^2\,dx$$

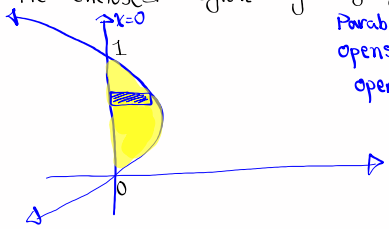
$$=\pi\int_1^5 (x-1)\,dx=\pi\left(\frac{x^2}{2}-x\right)\Big]_1^5=\pi\left[\left(\cancel{\frac{25}{2}}-5\right)-\left(\cancel{\frac{1}{2}}-1\right)\right]$$

$$=\pi\left[\frac{24}{2}-4\right]=\boxed{8\pi}$$

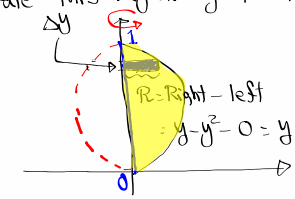
May 15-9:09 AM

Draw the enclosed region by $x = y - y^2$ and $x = 0$.

Parabola opens sideways
opens left



Rotate this region by Y-axis.



1) Is Ref. Rect. ⊥ A.O.R.? Yes

2) Is the enclosed region totally attached to A.O.R.? Yes

Disk Method

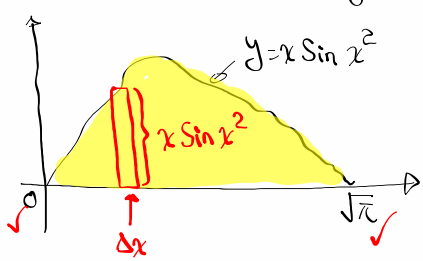
$$V = \int_c^d \pi R^2 \, dy = \int_0^1 \pi (y - y^2)^2 \, dy$$

$$= \pi \int_0^1 (y^2 - 2y^3 + y^4) \, dy = \pi \left[\frac{y^3}{3} - \frac{2y^4}{4} + \frac{y^5}{5} \right]_0^1$$

$$= \pi \left(\frac{1}{3} - \frac{1}{2} + \frac{1}{5} \right) = \pi \cdot \frac{10 - 15 + 6}{30} = \boxed{\frac{\pi}{30}}$$

May 15-9:17 AM

Consider the enclosed region below



at $x = 0$

$y = 0 \sin 0^2 = 0$

at $x = \sqrt{\pi}$

$y = \sqrt{\pi} \, \cancel{\sin \pi}^{\,0} = 0$

Find the area of this enclosed region

$$A = \int_0^{\sqrt{\pi}} x \sin x^2 \, dx$$

$u = x^2$

$du = 2x \, dx \rightarrow \frac{du}{2} = x \, dx$

$x = 0 \rightarrow u = 0^2 = 0$

$x = \sqrt{\pi} \rightarrow u = (\sqrt{\pi})^2 = \pi$

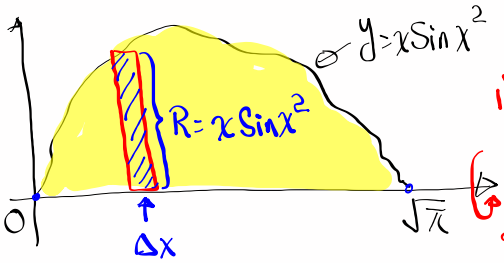
$$= \int_0^{\pi} \sin u \, \frac{du}{2} = \frac{1}{2} (-\cos u) \Big]_0^{\pi}$$

$$= \frac{1}{2} \left[\cos \pi - \cos 0 \right]$$

$$= \frac{1}{2} (-1 - 1) = \boxed{1}$$

May 15-9:27 AM

Rotate that region by x-axis, and Set-up the integral for the volume generated.



1) is Ref. Rect. $\perp$ A.O.R.? Yes

2) Is region completely attached to A.O.R.? Yes

Disk

$$V = \int_0^{\sqrt{\pi}} \pi \left(x \sin x^2\right)^2 dx$$

$$= \pi \int_0^{\sqrt{\pi}} x^2 \sin^2 x^2 \, dx$$

May 15-9:34 AM

If $f(x)$ is cont. on $[a,b]$, then the length of the curve $y = f(x)$ from $x=a$ to $x=b$ is

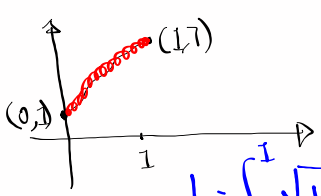
$$L = \int_a^b \sqrt{1 + [f'(x)]^2} \, dx$$

Find the arc length of the curve $y = 1 + 6x^{3/2}$ from $x=0$ to $x=1$.

$f(x) = 1 + 6x^{3/2}$

$f'(x) = 6 \cdot \frac{3}{2} x^{1/2} = 9\sqrt{x}$

$1 + [f'(x)]^2 = 1 + 81x$



$$L = \int_0^1 \sqrt{1 + 81x} \, dx$$

May 15-9:41 AM

$$L = \int_0^1 \sqrt{1+81x}\,dx$$

$u = 1 + 81x$

$du = 81\,dx \qquad \frac{du}{81} = dx$

$x = 0 \rightarrow u = 1$

$x = 1 \rightarrow u = 82$

$$= \int_1^{82} \sqrt{u} \cdot \frac{du}{81}$$

$$= \frac{1}{81}\int_1^{82} u^{1/2}\,du = \frac{1}{81} \cdot \frac{u^{3/2}}{3/2}\Bigg]_1^{82}$$

$$= \frac{2}{243}\left(u\sqrt{u}\right)\Bigg]_1^{82} = \frac{2}{243}\left(82\sqrt{82} - 1\right)$$

$$= \boxed{\frac{164\sqrt{82} - 2}{243}}$$

May 15-9:48 AM